Chapter 4 practice questions

1. **(a)** $x^4 = 16 \rightarrow x = \sqrt[4]{16} = 2$

(b) $3^x = 27 \rightarrow 3^x = 3^3 \rightarrow x = 3$

(c) $\log_8 x = -\frac{1}{3} \rightarrow 8^{-\frac{1}{3}} = x \rightarrow x = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

(d) $\log_6 (x-1) + \log_6 x = 1$ with domain $x > 1$

$\log_6 x(x-1) = \log_6 6 \implies x(x-1) - 6 = 0$

$x^2 - x - 6 = 0 \implies (x-3)(x+2) = 0$

$x = 3$ or $x = -2$, but only $x = 3$ is acceptable.

2. **(a)** $4^x = 36 \rightarrow x = \log_4 36 \approx 2.58$

(b) $5 \cdot 3^x = 18 \rightarrow 3^x = \frac{18}{5} \rightarrow x = \log_3 \frac{18}{5} \approx 1.17$

(c) $8^{-x} = \left(\frac{1}{4}\right)^3$; for this, it is convenient to reduce both sides to powers with base 2.

$\left(2^3\right)^{-x} = \left(\frac{1}{2^2}\right)3 \rightarrow 2^{3x} = 2^{-6} \rightarrow 3x = 6 \rightarrow x = 2$

(d) $6^x = 0.25^{2x-1}$; for this, the exponential functions cannot be reduced to a common base.

$\ln 6^x = \ln 0.25^{2x-1} \implies x \ln 6 = (2x-1)\ln 0.25$

$x(\ln 6 - 2\ln 0.25) = -\ln 0.25 \rightarrow x = \frac{-\ln 0.25}{\ln 6 - 2\ln 0.25} \approx 0.304$

3. **(a)** $\log_2 x^2 - \log_2 x + \log_2 3^2 = \log_2 \frac{x^2 3^2}{x} = \log_2 9x$

(b) $\ln 3 + \frac{1}{2}\ln(x-4) - \ln x = \ln 3 + \ln\sqrt{x-4} - \ln x = \ln \frac{3\sqrt{x-4}}{x}$

4. **(a)** $\log_b N = \frac{1}{2} \cdot 2 \cdot \log_b N = \frac{1}{2}\log_b N^2 = \frac{1}{2}3.78 = 1.89$

(b) $\log_b \frac{N^4}{\sqrt{M}} = \log_b N^4 - \log_b \sqrt{M} = \log_b \left(N^2\right)^2 - \log_b M^{\frac{1}{2}} = 2\log_b N^2 - \frac{1}{2}\log_b M =$

$= 2 \cdot 3.78 - \frac{1}{2} \cdot 5.42 = 4.85$

5. **(a)** Pablo's investment after t years is given by $M(t) = 2000 \cdot (1 + 6.75\%)^t$, so after 4 years its value is $M(4) = 2000(1.0675)^4 = €2597.18 \approx €2597$.

(b) This occurs when $1.0675^t = 2$, so for $t = \log_{1.0675} 2 \approx 10.6$ years, so after at least 11 years.

(c) This condition amounts to $(1+r)^{10} = 2$, which gives $1 + r = \sqrt[10]{2}$ or $r = \sqrt[10]{2} - 1 \approx 0.0718 = 7.18\%$

6. **(a)** for the first part of the deposit the account value is given by $A(t) = 1000 \cdot (1.04)^t$. At the end of the third year, the value of the account is $A(3) = 1000 \cdot 1.04^3 = \$1124.864$. This amount becomes the initial amount for the next four years, when the account value becomes $A(t) = 1124.864 \cdot (1 + 7\%)^t$. At the end of the fourth year, the value is $A(4) = 1124.864 \cdot 1.07^4 = \$1474.47$.

(b) If one single rate r has to account for this value, then $1474.47 = 1000 \cdot (1+r)^7$. Solving for r gives $(1+r)^7 = 1.47447 \to r = \sqrt[7]{1.47447} - 1 \approx 0.0570 = 5.7\%$.

7. **(a)** $\log_2 5 \cdot \log_5 2 = \log_2 5 \cdot \dfrac{\log_2 2}{\log_2 5} = \dfrac{\log_2 5}{\log_2 5} \cdot \log_2 2 = 1$

(b) $\log_4 8 = \dfrac{\log_2 8}{\log_2 4} = \dfrac{3}{2}$

(c) $4^{\log_2 6} = \left(2^2\right)^{\log_2 6} = 2^{2\log_2 6} = 2^{\log_2 6^2} = 6^2 = 36$

8. **(a)** This means finding $E(6) = 500(1.032)^6 \approx 604$

(b) The condition set is the inequality $E(t) > 750 \to 500(1.032)^t > 750$. Solving it yields $1.032^t > 1.5$, or $t > \log_{1.032} 1.5 \approx 12.87$. It is going to take 13 full years before the number of elephants exceeds 750.

9. **(a)** $\dfrac{22000}{25000} = 88\%$

(b) The value of the car after t years would be given by $C(t) = 25000 \cdot 0.88^t$, so after six years $C(6) = 25000 \cdot 0.88^6 \approx \$11610$.

(c) This would be equivalent to $C(t) < 8000 \to 25000 \cdot 0.88^t < 8000$.

Solving this gives $0.88^t < \frac{8000}{25000} \implies 0.88^t < 0.32$.

Taking logarithms base 0.88 of both sides gives $t > \log_{0.88} 0.32 = 8.91 \approx 9$ since $\log_{0.88} x$ is a decreasing function.

The car value drops below \$8000 in 2011.

10. **(a)** The domain is the set of real numbers; the range is the set of positive real numbers.

(b) The graph of f is the graph of e^x shifted two units to the right. Therefore, it has a horizontal asymptote $y = 0$ and no vertical asymptotes. The y-intercept is given by $(0, f(0)) = (0, \mathrm{e}^{-2}) = \left(0, \frac{1}{\mathrm{e}^2}\right)$.

(c)

$$y = \mathrm{e}^{x-2}$$
$$x = \mathrm{e}^{y-2}$$
$$\ln x = \ln \mathrm{e}^{y-2} = y - 2$$
$$y = \ln x + 2$$
$$f^{-1}(x) = \ln x + 2$$

(d) The domain is the set of positive real numbers; the range is the set of real numbers.

11. The population after t months is $P(t) = 500 \cdot (1 + 6\%)^t$.

(a) $P(4) = 500 \cdot 1.06^4 \approx 631$

(b) $P(4+12) = P(16) = 500 \cdot 1.06^{16} \approx 1270$

(c) **(i)** $A_0 = 500$

(ii) $b = 1.06$

(d) Setting $500 \cdot 1.06^t = 500 \cdot \mathrm{e}^{kt}$, we have $1.06^t = \mathrm{e}^{kt} \to (1.06)^t = (\mathrm{e}^k)^t$, from which we obtain $1.06 = \mathrm{e}^k \to k = \ln 1.06 \approx 0.05827$.

12. **(a)** **(i)** The argument of log must be positive, so $\frac{x}{x-2} > 0$, and the denominator can't be zero, so $x - 2 \neq 0$. Therefore, **either** both numerator and denominator are positive **or** both numerator and denominator are negative, in such a way that their ratio is positive: $x < 0$ **or** $x > 2$. Both also satisfy $x - 2 \neq 0$

(ii) In this case, both arguments have to be positive simultaneously, so $x > 0$ **and** $x > 2$. Both conditions are satisfied for $x > 2$ only.

(b) **(i)** $\log \frac{x}{x-2} = -2$

$\frac{x}{x-2} = 10^{-2}$

$\frac{x - 10^{-2}(x-2)}{x-2} = 0 \rightarrow x - 10^{-2}(x-2) = 0$

$x - 0.01x + 0.02 = 0 \rightarrow 0.99x = -0.02$

$x = -\frac{0.02}{0.99} = -\frac{2}{99}$, which is in the domain of the equation, so is acceptable.

(ii) $\log x - \log(x-2) = -2$

$\log \frac{x}{x-2} = -2$ and from here onwards it is the same as above but

$x = -\frac{2}{99}$ is not in the domain of this equation, so there are no solutions. In fact, since $\log x$ is an increasing function, $\log x > \log(x-2)$ and therefore $\log x - \log(x-2)$ is positive and cannot be equal to -2.

13. **(a)** Plugging in the given conditions $A(0) = 5000$ and $A(22) = 17000$ in the form $A(t) = Ce^{kt}$ gives two equations in C and k, $5000 = C \cdot e^{k \cdot 0}$ and $17000 = C \cdot e^{k \cdot 22}$.

The former gives $C = 5000$, so the latter becomes $e^{22k} = \frac{17000}{5000}$, or $22k = \ln\frac{17}{5}$.

Finally, $k = \frac{1}{22}\ln\frac{17}{5} \approx 0.0556$.

(b) Replacing the values of the parameters $C = 5000$ and $k = 0.0556$, and evaluating $C(60)$ gives $C(60) = 5000 \cdot e^{0.0556 \cdot 60} \approx 140532$.

14. **(a)** P is found by setting $y = 0$, so $2 - \log_3(x+1) = 0$. Solving for x gives $\log_3(x+1) = 2 \rightarrow x + 1 = 3^2 \rightarrow x = 8$

(b) Q is found by setting $x = 0$. Replacing gives $y = 2 - \log_3 1 = 2 - 0 = 2$.

(c) R is found by setting $y = 3$, so $2 - \log_3(x+1) = 3$. Solving for x gives

$\log_3(x+1) = -1 \rightarrow x + 1 = 3^{-1} \rightarrow x = \frac{1}{3} - 1 = -\frac{2}{3}$, so $R\left(-\frac{2}{3}, 3\right)$.

15. $\log_2(5x^2-x-2)=2+2\log_2 x \Rightarrow \log_2(5x^2-x-2)=\log_2 4+\log_2 x^2 \Rightarrow$

$\log_2(5x^2-x-2)=\log_2 4x^2 \Rightarrow \log_2(5x^2-x-2)-\log_2 4x^2=0 \Rightarrow \log_2\dfrac{5x^2-x-2}{4x^2}=0$

$\log_2\dfrac{5x^2-x-2}{4x^2}=0 \Rightarrow \dfrac{5x^2-x-2}{4x^2}=1 \Rightarrow \dfrac{5x^2-x-2-4x^2}{4x^2}=0\Rightarrow$

$x^2-x-2=0 \Rightarrow (x-2)(x+1)=0$

$x=2$ or $x=-1$. Only $x=2$ is acceptable as $x=-1$ is not in the domain of the right-hand side of the original equation.

16. $\log_2 4\sqrt{2}=x$, $\log_z y=4$ and $y=4x^2-2x-6+z$. First of all, evaluating *x* gives

$x=\log_2 4\sqrt{2}=\log_2 2^{\frac{5}{2}}=\dfrac{5}{2}$ so that the third condition becomes

$y=4\left(\dfrac{5}{2}\right)^2-2\cdot\dfrac{5}{2}-6+z=25-5-6+z=14+z$. Solving the second condition for *y* we have

$z^4=y$, and combining the second and third conditions gives

$z^4=14+z$. Solving with a GDC, or by inspection, we find that $z=2$ is a solution. So $y=14+2=16$. The other solution for z is negative and must be rejected because in the second condition z is the base of a logarithm and it cannot be negative.

17. **(a)** $\log_3 x-4\log_x 3+3=0$. Changing base to the second logarithm we have

$\log_3 x-4\dfrac{\log_3 3}{\log_3 x}+3=0$. Introducing $y=\log_3 x$ we have

$y-4\dfrac{1}{y}+3=0 \Rightarrow \dfrac{y^2-4+3y}{y}=0 \Rightarrow y^2+3y-4=0 \Rightarrow (y-1)(y+4)=0$

So $y=1$ or $y=-4$. Solving for *x* gives $\log_3 x=1\to x=3$ or

$\log_3 x=-4\to x=3^{-4}=\dfrac{1}{81}$.

Both solutions are acceptable as can be checked in the original equation.

(e) $\log_2(x-5)+\log_2(x+2)=3$. The domain is $x>5\cap x>-2\to x>5$.

$\log_2(x-5)(x+2)=\log_2 8$. The domain is $(x-5)(x+2)>0\to x<-2\cup x>5$, so if we find solutions in the domain of the second equation that are not in the domain of the first, that is, for $x<-2$, they have to be rejected.

Solving for *x* gives

$(x-5)(x+2)=8$

$x^2-3x-10-8=0\to x^2-3x-18=0$

$(x-6)(x+3)=0\to x=6$ or $x=-3$.

The latter solution has to be discarded, so the solution is $x=6$.

18. **(a)** $2\log a + 3\log b - \log c = \log a^2 + \log b^3 - \log c = \log \dfrac{a^2b^3}{c}$

(b) $3\ln x - \dfrac{1}{2}\ln y + 1 = \ln x^3 - \ln y^{\frac{1}{2}} + \ln \mathrm{e} = \ln \dfrac{\mathrm{e}\cdot x^3}{\sqrt{y}}$

19. If the activity is 79% of its initial value A_0, then $79\%A_0 = A_0\mathrm{e}^{-0.000124t} \to 0.79 = \mathrm{e}^{-0.000124t}$. Solving for t

$\ln 0.79 = -0.000124t$

$t = \dfrac{\ln 0.79}{-0.000124} \sim 1900.98 \approx 1900$ years.

20. The given condition means $\log_3(2c-3) - 4 = 0$. Solving for c

$\log_3(2c-3) = \log_3 3^4$

$2c - 3 = 81 \to c = \dfrac{81+3}{2} = 42$.

21. Setting $y = \ln x$ the equation becomes

$2y^2 = 3y - 1 \Rightarrow 2y^2 - 3y + 1 = 0$

$y = \dfrac{3 \pm \sqrt{9-8}}{4} = \dfrac{3\pm 1}{4} = 1$ or $\dfrac{1}{2}$.

Solving for x gives

$\ln x = 1 \to x = \mathrm{e}$ or

$\ln x = \dfrac{1}{2} \to x = \mathrm{e}^{\frac{1}{2}} = \sqrt{2}$. Both solutions are acceptable.

22. The investment after t years is given by $A(t) = 100\cdot(1+5\%)^t$.

(a) $V = A(20) = 100\cdot 1.05^{20} \approx \$265.33$

(b) The condition is $100\cdot\left(1+\dfrac{5}{12}\%\right)^{12t} > 265.33$, which gives

$\left(\dfrac{12.05}{12}\right)^{12t} > 2.6533$

$12t > \log_{\frac{12.05}{12}} 2.6533$

$t > \dfrac{1}{12}\log_{\frac{12.05}{12}} 2.6533 \approx 19.6$ years ≈ 235 months.

23. $9\log_5 x = 25\log_x 5$. Changing base,

$9\log_5 x = 25\dfrac{\log_5 5}{\log_5 x}$. Setting $y = \log_5 x$,

$9y = 25\dfrac{1}{y}$

$y^2 = \dfrac{25}{9} \rightarrow y = \pm\dfrac{5}{3}$. Solving for x

$\log_5 x = \dfrac{5}{3} \rightarrow x = 5^{\frac{5}{3}}$ or

$\log_5 x = -\dfrac{5}{3} \rightarrow x = 5^{-\frac{5}{3}}$. Both solutions are acceptable.

24. If the number of bacteria doubles every twenty minutes, it follows that $\mathrm{e}^{k\cdot 20} = 2$.

Solving for k

$20k = \ln 2 \Rightarrow k = \dfrac{\ln 2}{20}$